Int. J. Heat Mass Transfer. Vol. 27, No. 3, pp. 433 441, 1984
Printed in Great Britain

0017 9310/84 $3.00 + 0.00
Pergamon Press Ltd.

NATURAL CONVECTION HEAT TRANSFER TO LIQUID METALS BELOW DOWNWARD FACING HORIZONTAL SURFACES

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(*Received* 15 *December* 1982)

Abstract—For special applications to liquid metals two-dimensional natural convection below an infinite strip and axisymmetric convection below a circular plate are calculated for a uniform surface temperature as well as for a uniform surface heat flux. The laminar boundary-layer equations are solved iteratively, starting with a forced convection of an unknown upstream velocity, which is determined with potential theory. Locally similar solutions are obtained for velocity and temperature profiles. Finally, a heat balance fixes the upstream velocity with respect to buoyancy forces. Thus, a local Nusselt number is correlated with the Grashof and Prandtl numbers; it is predicted to be higher than those previously achieved with integral methods but agrees better with experiments.

NOMENCLATURE

g	gravitational acceleration
m	$n-1$
n	$n = 0$ for two-dimensional convection, $n = 1$ for axisymmetric convection
p	pressure
q_w	surface heat flux, $-(\lambda \partial T/\partial z)_w$
r	horizontal coordinate
R	half-width of the infinite strip or radius of the circular plate
T	temperature
T_w, T_∞	surface temperature and ambient temperature, respectively
u	r-direction velocity component
v	vertical velocity component
v_∞	upstream velocity (unknown)
z	vertical coordinate.

Greek symbols

α	thermal expansion coefficient
κ	thermal diffusivity
λ	thermal conductivity
ν	kinematic viscosity
ψ	stream function
Ω	vorticity, $\partial u/\partial z - \partial v/\partial r$.

Dimensionless quantities

A	$Pe^{5/2}/Gr\ Pr^2$ for specified surface temperature, $Pe^3/Gr^*\ Pr^2$ for specified surface heat flux
Gr	Grashof number, $\alpha g(T_w - T_\infty)R^3/\nu^2$
Gr^*	modified Grashof number, $\alpha g q_w R^4/(\lambda \nu^2)$
M	stream function, $\psi/(v_\infty R)$
$M^{(1)}$	iterated stream function
Nu	Nusselt number, $q_w R/\lambda(T_w - T_\infty)$
Pe	Péclet number, $v_\infty R/\kappa$
Pr	Prandtl number, ν/κ
U	profile of the horizontal velocity, $-\int_\eta^\infty \omega_1 \,d\eta$
V	profile of the vertical velocity, $\int_0^\eta U \,d\eta$
η	vertical coordinate, $z\sqrt{Pe}/R$
ω	vorticity, $R^2 A\Omega/\kappa\ Pe^{3/2}$
ω_1	profile of the vorticity, $\omega(\xi,\eta) = \xi \cdot \omega_1(\eta)$
θ	temperature: $(T-T_\infty)/(T_w-T_\infty)$ for specified surface temperature; $\sqrt{Pe}\,(T-T_\infty)/q_w R$ for specified surface heat flux
θ_0, θ_1	η-dependent coefficients of temperature, $\theta(\xi,\eta) = \theta_0(\eta) + \xi^2\theta_1(\eta)$
$\theta^{(1)}$	iterated temperature profile
ξ	horizontal coordinate, r/R.

1. INTRODUCTION

ALTHOUGH meltdown of the nuclear core of a liquid metal cooled fast breeder reactor is extremely improbable, current designs include an internal core catcher in the lower plenum of the reactor vessel to assure post accident heat removal and the integrity of the primary vessel. Some of the proposed core catcher designs consist of a flat plate which is either horizontal or near-horizontal. Should a meltdown accident occur, core debris may settle on this core catcher to form a debris bed. As part of the decay heat will penetrate the plate, a pool of liquid metal underneath the core catcher must remove heat by natural convection [1]. In order to estimate the temperature distribution below the plate it is essential to calculate the heat transfer coefficient due to natural convection at low Prandtl numbers.

Previous studies of downward heat transfer by convection are based on the well-known boundary-layer equations. Since Gill *et al.* [2] pointed out that a similarity solution cannot be achieved with current methods as had been obtained for inclined or upward facing flat plates, natural convection has been analysed approximately with several variations of an integral method.

This treatment was first used by Singh *et al.* [3] to solve free convection at a Prandtl number near unity for an isothermal infinite strip, a circular plate and a square plate facing downward. An important problem in all these integral methods was to find a boundary condition for the boundary-layer thickness at the edges of the plate; it was wrongly approximated to zero by Singh *et al.* [3].

To obtain better agreement with the experiments of Saunders *et al.* [4], Singh and Birkebak [5] revised the analysis by assuming the first horizontal derivative of the boundary layer to be infinite at the edges of the plate, which results in a finite thickness there. The method was applied to an isothermal infinite strip and was extended to more general Prandtl numbers, $0.025 \leqslant Pr < \infty$.

Furthermore, Clifton and Chapman [6] tried to solve this problem by introducing a critical thickness at the edges of the plate, which was obtained from hydraulic flow theory.

As the region around the edges of the plate seemed to be sensitive to total heat transfer, Birkebak and Abdulkadir [7] investigated experimentally the convection under a horizontal square plate in water. The measured heat transfer was in good agreement with theory [3], except at the edges of the plate.

Velocity and temperature profiles below a quasi-infinite strip in air were measured by Aihara *et al.* [8]. Although these profiles did not coincide with those presumed in the integral method [5], local heat transfer again agreed well with theory, which proves that the calculated heat transfer does not depend sensitively on these profiles.

Fujii *et al.* [9] calculated heat transfer correlations for an infinite strip, a round and a square plate with given heat flux, and for a large range of Prandtl numbers, $0.001 \leqslant Pr < \infty$, also including those of liquid metals. They assumed the same boundary-layer thickness at the edges of the plate as Singh and Birkebak [5]; the temperature and velocity profiles were the same as in ref. [8]. Although these relations can well be applied to fluids with moderate Prandtl numbers and have been verified recently by Hatfield and Edwards [10] even for oil with a high Prandtl number, no conclusions can be drawn concerning the validity of these relations at very low Prandtl numbers as neither velocity and temperature profiles nor the boundary condition at the edges of the plate agree with experiments. Heat transfer coefficients recently measured by Sherriff and Davies [11] under a quasi-infinite strip in liquid sodium are about 20% higher than predicted by Fujii *et al.*, which gives reason for further doubts.

Therefore, it became the objective of the work reported here to reconsider the heat transfer from downward facing flat surfaces, especially at low Prandtl numbers. To circumvent problems with unknown profiles and with an unknown behaviour of the boundary-layer thickness at the edges of the plate, a completely different method was employed.

2. FORMULATION OF THE PROBLEM

As mentioned above, similarity solutions of the boundary-layer equations exist for inclined or upward facing flat plates, but have still not been obtained for a downward facing plate.

A plausible reason for these difficulties is that under a horizontal flat plate natural convection occurs because of the finite length of the plate; it is induced by the motion around the edges of the plate and therefore by an event that occurs downstream. As the boundary-layer equations are parabolic, they only take into account the influence of upstream events, so it should be impossible to solve this elliptic problem with the boundary-layer equations alone. This consideration also explains why the integral methods call for a boundary condition at the edges of the plate, which actually does not exist. With such an artificial condition some of those terms had to be substituted, which had been cancelled without justification by the boundary-layer approximations.

The following procedure is to be considered as a first step in an iteration procedure. It presumes a potential flow around the plate, which is the solution of an elliptic problem, and it results in an analytic solution of the boundary-layer equations with locally similar velocity and temperature profiles in a region around the stagnation point at the centre of the plate.

3. ANALYSIS

The governing equations for laminar steady-state natural convection using the Boussinesq approximation, and without boundary-layer approximations (Fig. 1) are

$$\frac{\partial u}{\partial r} + n\frac{u}{r} + \frac{\partial v}{\partial z} = 0,$$

$$u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial^2 u}{\partial r^2} + n\frac{\partial}{\partial r}\left(\frac{u}{r}\right) + \frac{\partial^2 u}{\partial z^2}\right],$$

$$u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 v}{\partial r^2} + n\frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right] - \alpha g(T - T_\infty),$$

$$u\frac{\partial T}{\partial r} + v\frac{\partial T}{\partial z} = \kappa\left[\frac{\partial^2 T}{\partial r^2} + n\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right], \quad (1)$$

where $n = 0$ for two-dimensional (2-D) convection below an infinite strip, and $n = 1$ for axisymmetric convection below a circular plate.

If a stream function, ψ, and a vorticity, Ω, are defined such that

$$u = \frac{\partial \psi}{\partial z}, \quad v = -\frac{\partial \psi}{\partial r} - n\frac{\psi}{r}, \quad \Omega = \frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}, \quad (2)$$

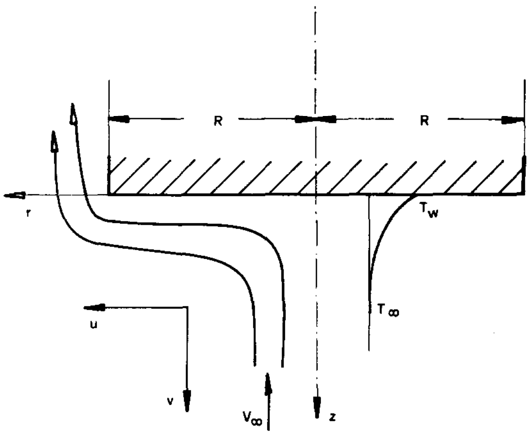

FIG. 1. Natural convection below a horizontal plate.

then equations (1) become

$$\frac{1}{Pe}\left[\frac{\partial^2 M}{\partial \xi^2}+n\frac{\partial}{\partial \xi}\left(\frac{M}{\xi}\right)\right]+\frac{\partial^2 M}{\partial \eta^2}=\frac{\omega}{A\sqrt{Pe}},$$

$$\frac{\partial M}{\partial \eta}\left(\frac{\partial \omega}{\partial \xi}-n\frac{\omega}{\xi}\right)-\left(\frac{\partial M}{\partial \xi}+n\frac{M}{\xi}\right)\frac{\partial \omega}{\partial \eta}$$
$$=\frac{Pr}{\sqrt{Pe}}\left\{\frac{1}{Pe}\left[\frac{\partial^2 \omega}{\partial \xi^2}+n\frac{\partial}{\partial \xi}\left(\frac{\omega}{\xi}\right)\right]+\frac{\partial^2 \omega}{\partial \eta^2}\right\}+\frac{1}{\sqrt{Pe}}\frac{\partial \theta}{\partial \xi},$$

$$\frac{\partial M}{\partial \eta}\frac{\partial \theta}{\partial \xi}-\left(\frac{\partial M}{\partial \xi}+n\frac{M}{\xi}\right)\frac{\partial \theta}{\partial \eta}$$
$$=\frac{1}{\sqrt{Pe}}\left\{\frac{1}{Pe}\left[\frac{\partial^2 \theta}{\partial \xi^2}+n\frac{1}{\xi}\frac{\partial \theta}{\partial \xi}\right]+\frac{\partial^2 \theta}{\partial \eta^2}\right\}, \quad (3)$$

when the equations are made dimensionless with

$$\xi=\frac{r}{R}, \qquad \eta=\frac{z}{R}\sqrt{Pe},$$
$$M=\frac{\psi}{v_\infty R}, \qquad \omega=\frac{R^2}{\kappa}\frac{A}{Pe^{3/2}}\Omega,$$
$$Pe=\frac{v_\infty R}{\kappa}, \qquad Pr=\frac{\nu}{\kappa}, \quad (4a)$$
$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, \quad A=\frac{Pe^{5/2}}{Gr\, Pr^2},$$
$$Gr=\frac{\alpha g(T_w-T_\infty)R^3}{\nu^2},$$

for a specified surface temperature, or

$$\theta=\sqrt{Pe}\frac{(T-T_\infty)\lambda}{q_w R}, \quad A=\frac{Pe^3}{Gr^*\, Pr^2},$$
$$Gr^*=\frac{\alpha g q_w R^4}{\lambda \nu^2}, \quad (4b)$$

for a specified surface heat flux.

In this representation fluid motion depends on three dimensionless quantities: a Péclet number, Pe, which includes an unknown upstream velocity, v_∞, perpendicular to the plate, a Grashof number, Gr or Gr^*, which includes buoyancy forces and the Prandtl number, Pr. As the upstream flow will be driven only by buoyancy forces, there must be a connection between Pe and Gr or Gr^*, respectively. The iterative computation will first treat both dimensionless quantities separately; the unknown connection will finally be given by an energy balance which fixes the dimensionless group A introduced in equations (4a) and (4b).

At high Péclet numbers fluid motion can be divided into an outer flow which can be determined by means of the potential theory, and a boundary flow which can be computed employing boundary-layer approximations. Thus, the outer flow is a solution of

$$\frac{1}{Pe}\left[\frac{\partial^2 M}{\partial \xi^2}+n\frac{\partial}{\partial \xi}\left(\frac{M}{\xi}\right)\right]+\frac{\partial^2 M}{\partial \eta^2}=0, \quad (5)$$

with the dimensionless boundary conditions

$$\eta=0: \quad M=0,$$
$$\eta\to\infty: \quad \frac{\partial M}{\partial \xi}+n\frac{M}{\xi}=1. \quad (6)$$

Close to the surface the governing equations are

$$\frac{\partial M}{\partial \eta}\frac{\partial \theta}{\partial \xi}-\left(\frac{\partial M}{\partial \xi}+n\frac{M}{\xi}\right)\frac{\partial \theta}{\partial \eta}=\frac{1}{\sqrt{Pe}}\frac{\partial^2 \theta}{\partial \eta^2}, \quad (7)$$

$$\frac{\partial M}{\partial \eta}\left(\frac{\partial \omega}{\partial \xi}-n\frac{\omega}{\xi}\right)-\left(\frac{\partial M}{\partial \xi}+n\frac{M}{\xi}\right)\frac{\partial \omega}{\partial \eta}$$
$$=\frac{1}{\sqrt{Pe}}\left[Pr\frac{\partial^2 \omega}{\partial \eta^2}+\frac{\partial \theta}{\partial \xi}\right], \quad (8)$$

$$\frac{\partial^2 M}{\partial \eta^2}=\frac{\omega}{A\sqrt{Pe}}. \quad (9)$$

The dimensionless boundary conditions are

$\eta=0$: $M=0$,

$\omega=0$,

$\theta=1$ for a specified surface temperature

or

$\partial\theta/\partial\eta=-1$ for a specified surface heat flux, (10a)

$\eta\to\infty$: $\partial M/\partial\eta=0$,

$\omega=0$,

$\theta=0$, (10b)

where $\omega=0$ at $\eta=0$ corresponds to a slip boundary which was taken as an approximation for fluid motion at low Prandtl numbers.

If only the region around the stagnation point is considered, the classical solutions of equation (5) for an infinite strip [12] and for a circular plate [13], arranged perpendicular to a stream, can be expanded in series to yield

$$M = \left(m+\frac{2}{\pi}n\right)\frac{1}{\sqrt{Pe}}\xi\eta(1+\tfrac{1}{2}\xi^2), \qquad (11)$$

with $m = n-1$.

The series have been truncated to two terms of ξ and one term of η, which is the lowest possible order to include the finite size of the plate.

Inserting this stream function, M, into the heat balance, equation (7), and assuming the temperature, θ, to be

$$\theta(\xi,\eta) = \theta_0(\eta)+\xi^2\theta_1(\eta),$$

equation (7) can be grouped like powers of ξ and can be converted to a system of two linear ordinary differential equations

$$\theta_0''+\left(m+\frac{4}{\pi}n\right)\eta\theta_0' = 0, \qquad (12)$$

$$\theta_1''+\left(m+\frac{4}{\pi}n\right)\eta\theta_1'-\left(2m+\frac{4}{\pi}n\right)\theta_1 = -\left(\frac{3}{2}m+\frac{4}{\pi}n\right)\eta\theta_0'. \qquad (13)$$

The solutions of equation (12)

$$\theta_0 = 1-\left(\sqrt{\left(\frac{2}{\pi}\right)}m+\frac{2\sqrt{2}}{\pi}n\right)\times\int_0^\eta \exp\left[-\left(\frac{m}{2}+\frac{2n}{\pi}\right)\eta^2\right]\mathrm{d}\eta, \qquad (14)$$

for the specified surface temperature and

$$\theta_0 = \sqrt{\left(\frac{\pi}{2}\right)}m+\frac{\pi}{2\sqrt{2}}n-\int_0^\eta \exp\left[-\left(\frac{m}{2}+\frac{2n}{\pi}\right)\eta^2\right]\mathrm{d}\eta, \qquad (15)$$

for the specified heat flux case are presented in Figs. 2(a) and 3(a).

Next θ_0 is inserted in equation (13), it has been solved numerically by means of a simple finite-difference method. Solutions of which are shown in Figs. 2(b) and 3(b).

If the stream function, M, and the temperature, θ, are substituted into the vorticity balance, equation (8), a one term power series of the vorticity

$$\omega(\xi,\eta) = \xi\cdot\omega_1(\eta),$$

leads to the ordinary differential equation

$$Pr\,\omega_1''+\left(m+\frac{4}{\pi}n\right)\eta\omega_1'-m\omega_1 = -2\theta_1, \qquad (16)$$

which has also been solved numerically for low Prandtl numbers, $Pr \leqslant 10^{-2}$. Because an asymptotic solution for a vanishing Prandtl number can be expected, only a slight dependence on Pr exists close to the surface, as demonstrated in Figs. 2(c) and 3(c) for $Pr = 10^{-2}$ and 10^{-3}.

With this vorticity, ω, a new stream function, $M^{(1)}$, can be computed, if

$$\frac{\partial^2 M^{(1)}}{\partial\eta^2} = \frac{\xi\omega_1(\eta)}{A\sqrt{Pe}} \qquad (17)$$

is integrated twice. A first integration of equation (17) yields

$$\frac{\partial M^{(1)}}{\partial\eta} = \frac{\xi}{A\sqrt{Pe}}U(Pr,\eta), \qquad (18)$$

where

$$U(Pr,\eta) = -\int_\eta^\infty \omega_1(Pr,\eta)\,\mathrm{d}\eta$$

is proportional to the horizontal component of the velocity. By further integration one obtains

$$M^{(1)} = \frac{\xi}{A\sqrt{Pe}}V(Pr,\eta), \qquad (19)$$

where

$$V(Pr,\eta) = \int_0^\eta U(Pr,\eta)\,\mathrm{d}\eta$$

is proportional to the vertical component of the velocity. Both U and V have been determined for $Pr = 10^{-2}$ and 10^{-3}. As can be seen in Figs. 2(d) and (e), and 3(d) and (e), the dependence on the Prandtl number of these quantities is negligible, if $Pr \leqslant 10^{-2}$.

The iteration can now be finished, if the new stream function, $M^{(1)}$, is chosen to be of the same order as the stream function, M, which we took as an initial guess from the potential theory. More strictly speaking, the total horizontal convective heat transport due to both stream functions, M and $M^{(1)}$, is set equal to

$$\int_0^\infty \theta_0\frac{\partial M}{\partial\eta}\,\mathrm{d}\eta = \int_0^\infty \theta_0\frac{\partial M^{(1)}}{\partial\eta}\,\mathrm{d}\eta. \qquad (20)$$

Regarding equations (11) and (18) one obtains

$$\left(m+\frac{2}{\pi}n\right)\frac{\xi}{\sqrt{Pe}}\int_0^\infty \theta_0(\eta)\,\mathrm{d}\eta = \frac{\xi}{A\sqrt{Pe}}\int_0^\infty \theta_0(\eta)U(Pr,\eta)\,\mathrm{d}\eta, \qquad (21)$$

thus the dimensionless group

$$A = \frac{Pe^{5/2}}{Gr\,Pr^2},$$

or

$$A = \frac{Pe^3}{Gr^*\,Pr^2},$$

respectively, can be fixed to a certain constant given in Table 1.

Table 1. Results for the dimensionless group A

A	Infinite strip	Circular plate
Specified surface temperature	0.188	0.294
Specified surface heat flux	0.274	0.411

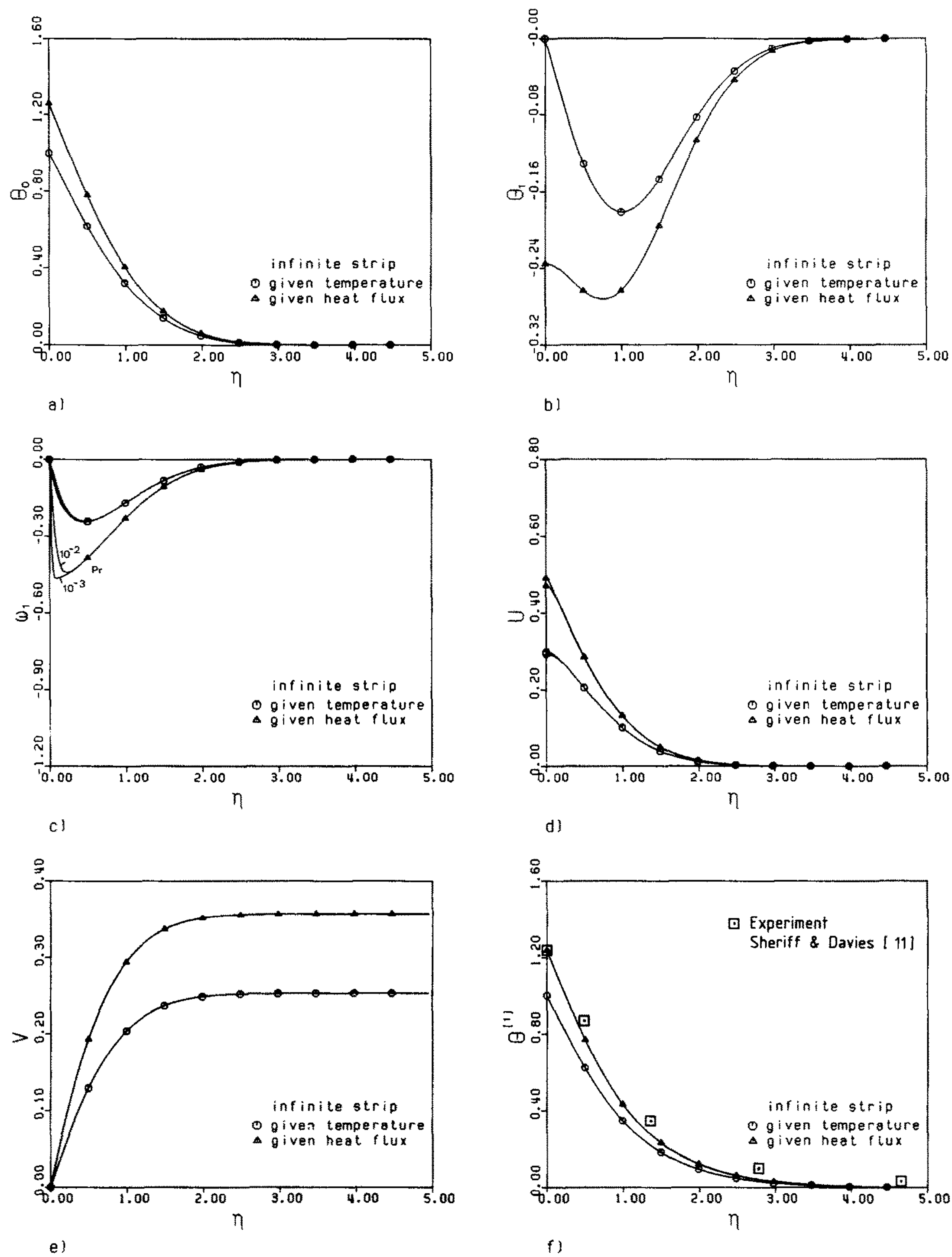

FIG. 2. Solutions for an infinite strip: (a) temperature coefficient θ_0; (b) temperature coefficient θ_1; (c) vorticity profile; (d) horizontal velocity profile; (e) vertical velocity profile; (f) iterated temperature profile.

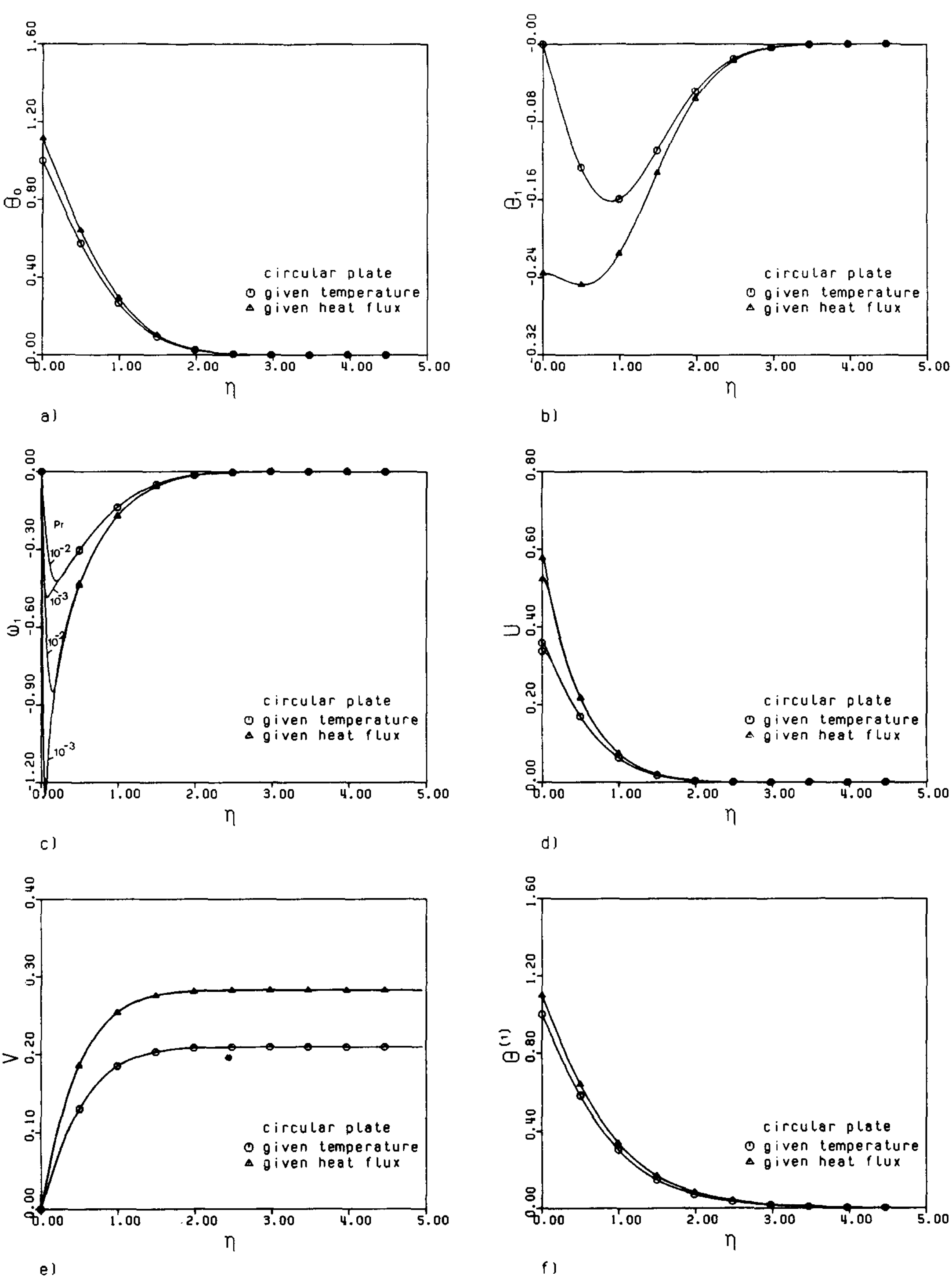

FIG. 3. Solutions for a circular plate: (a) temperature coefficient θ_0; (b) temperature coefficient θ_1; (c) vorticity profile; (d) horizontal velocity profile; (e) vertical velocity profile; (f) iterated temperature profile.

Therefore Pe can be replaced by $(A\ Gr\ Pr^2)^{2/5}$ and $(A\ Gr^*\ Pr^2)^{1/3}$, respectively, and can be inserted into the solutions of θ_0, equations (14) and (15), respectively, to give a correlation for the local Nusselt number

$$Nu = \frac{q_w R}{\lambda(T_w - T_\infty)} = -\sqrt{Pe}\frac{(\partial\theta/\partial\eta)_{\eta=0}}{\theta_{\eta=0}}, \quad (22)$$

in the vicinity of the stagnation point.

Results are

$$Nu = 0.571(Gr\ Pr^2)^{1/5}, \quad (23)$$

for an infinite strip with a specified surface temperature

$$Nu = 0.643(Gr^*\ Pr^2)^{1/6}, \quad (24)$$

for an infinite strip with a specified surface heat flux

$$Nu = 0.705(Gr\ Pr^2)^{1/5}, \quad (25)$$

for a circular plate with a specified surface temperature, and

$$Nu = 0.776(Gr^*\ Pr^2)^{1/6}, \quad (26)$$

for a circular plate with a specified surface heat flux.

To assure the accuracy of these correlations, one can also compute local Nusselt numbers by inserting the new stream function, $M^{(1)}$, into the heat balance, equation (7). If the resultant new temperature, $\theta^{(1)}$, is chosen to depend only on η, equation (7) yields

$$-(1+n)V(Pr,\eta)\frac{d\theta^{(1)}}{d\eta} = A\frac{d^2\theta^{(1)}}{d\eta^2}, \quad (27)$$

which can be integrated numerically when A is given in Table 1. Results are shown in Figs. 2(f) and 3(f), local Nusselt numbers differ by less than 2% from correlations (23)–(26). Also in Fig. 2(f) a temperature profile at the centre of an infinite strip is plotted, which was measured in liquid sodium by Sheriff and Davies [11]. The data agree with theory within the experimental error limits.

4. DISCUSSION

Two-dimensional convection below an infinite strip and axisymmetric convection below a circular plate have been computed for a uniform surface temperature and for a uniform heat flux.

Results are locally similar for solutions of the laminar boundary-layer equations which have been developed iteratively from solutions of the potential theory. They are valid within a region around the stagnation point at the centre of the plate. The local Nusselt number has been correlated with the Grashof number and the Prandtl number, provided that the Prandtl number is low, e.g. $Pr \leqslant 10^{-2}$, and that the Grashof number is sufficiently high. The latter restriction is essential to permit the boundary-layer approximations which have neglected terms of $1/Pe$ in equation (3). To obtain a Péclet number of at least 100 the Grashof number should be limited to $Gr^*\ Pr^2 \gtrsim 10^6$.

Previous correlations for an infinite strip with uniform surface heat flux, which have been determined by theory and experiments, can be compared with equation (24).

Fujii *et al.* [9] obtained with an integral method a Nusselt number which is 22% smaller, namely

$$Nu = 0.50(Gr^*\ Pr^2)^{1/6}.$$

Recent experiments by Sherriff and Davies [11] were correlated as

$$\overline{Nu} = 0.621(Gr^*\ Pr^2)^{1/6},$$

for the mean Nusselt number, and

$$Nu = 0.60(Gr^*\ Pr^2)^{1/6},$$

for the local Nusselt number at the stagnation point. The experimental error is estimated to be 8%. Preliminary results of Tenchine and Amblard [14] for

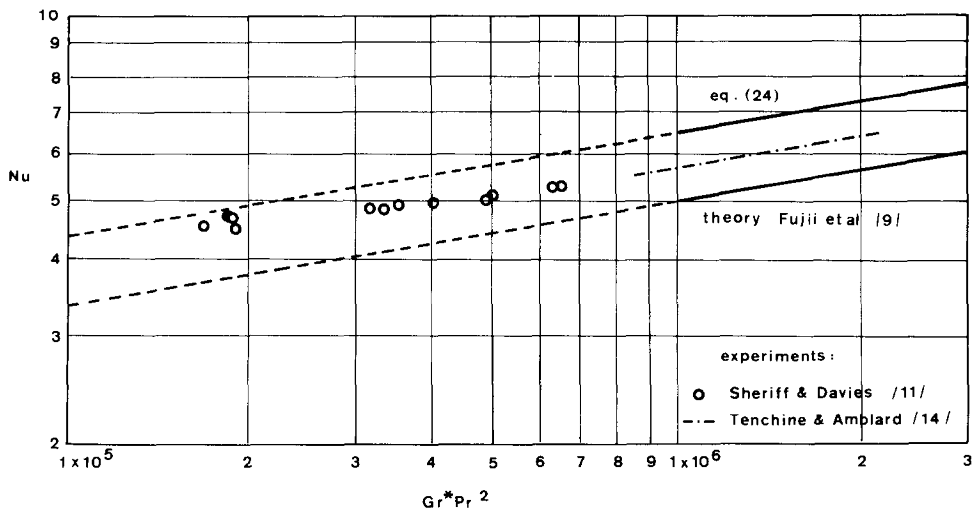

FIG. 4. Local Nusselt numbers at the stagnation point below an infinite strip with uniform heat flux.

higher Grashof numbers were correlated as

$$\overline{Nu} = 0.59(Gr^* \, Pr^2)^{1/6},$$

for the mean Nusselt number, which would be equivalent to

$$Nu = 0.57(Gr^* \, Pr^2)^{1/6},$$

for the local Nusselt number, according to ref. [11]. The experimental error is estimated to be 15%. These results are compared with equation (24) in Fig. 4.

In contrast to the results of the integral methods experiments agree with similarity solutions within the error bounds, even at lower Grashof numbers than permitted by this theory. However a tendency can be noticed that similarity solutions slightly overpredict the experimental results.

Acknowledgements—I am grateful to U. Müller for the proposal of this problem and for many valuable discussions.

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CONVECTION THERMIQUE NATURELLE DANS DES METAUX LIQUIDES SOUS DES SURFACES HORIZONTALES TOURNEES VERS LE BAS

Résumé—Pour des applications spéciales aux métaux liquides, la convection naturelle dimensionnelle sous une bande infinie et la convection axisymétrique sous une plaque circulaire sont étudiées pour une température pariétale uniforme et pour un flux uniforme sur la surface. Les équations de couche limite laminaire sont résolues par itération, partant avec une convection forcée de vitesse en amont inconnue et qui est déterminée à l'aide de la théorie potentielle. Des solutions localement similaires sont obtenues pour des profils de vitesse et de température. Finalement, un bilan d'énergie fixe la vitesse en amont en tenant compte dés forces d'Archimède. Ainsi, un nombre de Nusselt local est exprimé en fonction des nombres de Grashof et de Prandtl; il est supérieur à celui antérieurement obtenu par des méthodes intégrales, mais il s'accorde mieux avec les expériences.

WÄRMEÜBERTRAGUNG AN FLÜSSIGE METALLE AN DER UNTERSEITE EINER HORIZONTALEN FLÄCHE BEI NATÜRLICHER KONVEKTION

Zusammenfassung—Für die spezielle Anwendung bei Metallschmelzen wurden die zweidimensionale natürliche Konvektion unterhalb eines infiniten Streifens und die achsensymmetrische Konvektion unterhalb einer Kreisplatte für gleichförmige Wandtemperatur und für gleichförmige Wärmestromdichte berechnet. Die laminaren Grenzschichtgleichungen wurden iterativ gelöst. Dabei wurde mit erzwungener Konvektion bei zunächst unbekannter Aufwärts-Geschwindigkeit begonnen und diese dann mit der Potentialtheorie bestimmt. Für das örtliche Geschwindigkeits- und Temperaturprofil ergaben sich ähnliche Lösungen. Schließlich wurde die Aufwärts-Geschwindigkeit mit Hilfe einer Wärmebilanz an die Auftriebskräfte gekoppelt. Auf diese Weise wurde die örtliche Nußelt-Zahl mit der Grashof- und der Prandtl-Zahl korreliert; die Zahlen liegen höher als diejenigen, die sich bisher mit integralen Methoden ergaben, aber sie stimmen besser mit Versuchsergebnissen überein.

СВОБОДНО-КОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС К ЖИДКИМ МЕТАЛЛАМ ПОД ОБРАЩЕННЫМИ ВНИЗ ГОРИЗОНТАЛЬНЫМИ ПОВЕРХНОСТЯМИ

Аннотация—Рассчитаны двумерная естественная конвекция в жидких металлах под бесконечным участком и осесимметричная конвекция под круглой пластиной при однородной температуре поверхности, а также для однородного на поверхности теплового потока. Уравнения ламинарного пограничного слоя решаются методом итераций, начиная с вынужденной конвекции с неизвестной скоростью подъемного течения, которая определяется с помощью теории потенциала. Получаются автомодельные решения для профилей скорости и температуры. Наконец, тепловой баланс устанавливает скорость подъемного течения с учетом архимедовых сил. Таким образом, локальное число Нуссельта связано с числами Грасгофа и Прандтля; показано, что оно больше чисел, полученных ранее интегральными методами, и лучше согласуется с результатами экспериментов.